

Ch5.6 p1

MA121

June 13/8

exercises.

2p167 compute Δy from

$$y = e^{3x}, \quad x = \frac{1}{3}, \quad \Delta x = -0.95$$

$$\text{Formula } \Delta y \approx f'(x) \Delta x$$

$$\text{or } \Delta y \approx \frac{dy}{dx} \Delta x$$

So, from $y = e^{3x}$

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= (e^{3x})(3x)' \ln e \\ &= (e^{3x})(3)(1) \end{aligned}$$

$$\Rightarrow \frac{dy}{dx} = 3e^{3x}$$

$$\text{Thus, } \Delta y \approx (3e^{3x}) \Delta x$$

$$\Rightarrow \Delta y \approx 3e^{3(1/3)} (-0.95)$$

$$\Rightarrow \Delta y \approx -2.85e$$

$$\approx -7.747...$$

7p167

$$\ln 0.91 = ?$$

let $f(x) = \ln x$, let $x_1 = 0.91$ and $x_0 = 1$

we know exactly $\ln x_0 = \ln 1 = 0$ and $x_0 = 1$ is close to $x_1 = 0.91$. (choose x_0 such that we know the exact value of $f(x_0) = \ln x_0$ and x_0 is close to $x_1 = 0.91$)

$$f(x_1) \approx f(x_0) + f'(x_0)(x_1 - x_0)$$

$$\Rightarrow f(0.91) = \ln 0.91$$

$$\approx f(1) + f'(1)(0.91 - 1)$$

from $f(x) = \ln x$

$$\Rightarrow f'(x) = \frac{1}{x}$$

$$\Rightarrow f'(x_0) = f'(1) = \frac{1}{1} = 1$$

$$\therefore \ln 0.91 \approx \ln 1 + \frac{1}{1}(0.91 - 1) = 0 + (0.91 - 1) \approx -0.09$$

Ch 5.6 p2

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12 p 168

given $P = 1200 - 8q - 0.0005q^2$ $10 \leq q \leq 40$

we want to find Δp given $\Delta q = 2$ at the

Current $q_0 = 25$

$\Delta p \approx \frac{dP}{dq} \Delta q$ From $P = 1200 - 8q - 0.0005q^2$

$\Rightarrow \frac{dP}{dq} = -8 - 0.01q$ when $q_0 = 25$

$\frac{dP}{dq} = -8 - 0.01(25)$

$= -8 - 0.25$

$= -8.25$

so, $\Delta p \approx (-8.25)(2)$

$\Rightarrow \Delta p \approx -\16.50

$\therefore$ rent has to be lowered by \$16.50 per unit per month.

5.7

Elasticity of Demand.

When we increase the price, P , of an item, we expect a drop in demand, q .

Revenue is $R = pq$. Will we see a drop in revenue, or an increase?

PRICE

↑

↑

REVENUE

↑

↓

WHAT WE CALL IT

INELASTIC

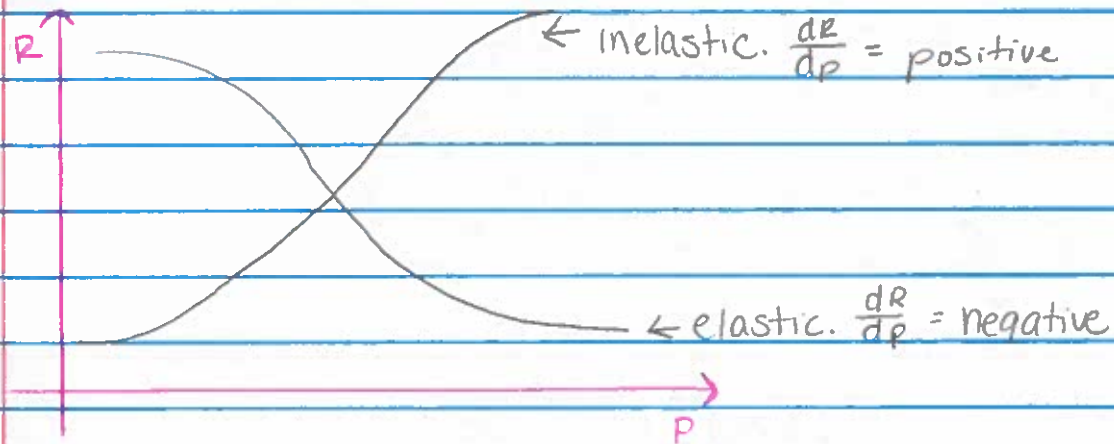
ELASTIC

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MAIZI

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Graphs of Revenue vs Price



We know that $R = pq$. We differentiate with respect to p .

$$\Rightarrow \frac{dR}{dp} = q \frac{dp}{dp} + p \frac{dq}{dp}$$

$$\Rightarrow \frac{dR}{dp} = q + p \frac{dq}{dp}$$

i) When demand is price elastic,

$$\frac{dR}{dp} < 0$$

$$\Rightarrow q + p \frac{dq}{dp} < 0$$

$$\Rightarrow -p \frac{dq}{dp} > q$$

$$\Rightarrow -\frac{p}{q} \frac{dq}{dp} > 1$$

ii) When demand is price inelastic

$$\frac{dR}{dp} > 0$$

$$\Rightarrow q + p \frac{dq}{dp} > 0$$

$$\Rightarrow -p \frac{dq}{dp} < q$$

$$\Rightarrow -\frac{p}{q} \frac{dq}{dp} < 1$$

Let's introduce a new symbol, η ("eta"), where

$\eta = -\frac{p}{q} \frac{dq}{dp}$ This defines price elasticity of the demand of an item.

In Summary

$$\eta = -\frac{P}{Q} \frac{dQ}{dP}$$

i) if $\eta > 1$, then demand is elastic.ii) if $\eta < 1$, then demand is inelastic

$$\text{From } \eta = -\frac{P}{Q} \frac{dQ}{dP}$$

$$\Rightarrow \eta = -\frac{P dQ}{Q dP} = -\frac{P dQ}{Q dP}$$

$$\Rightarrow \eta = -\frac{\frac{dQ}{Q}}{\frac{dP}{P}}$$

$$\Rightarrow \eta = -\frac{\frac{dQ}{Q}}{\frac{dP}{P}} \quad \text{where } \frac{dQ}{Q} = \% \text{ drop in demand}$$

and $\frac{dP}{P} = \% \text{ drop in price}$

Recall that in a demand function,

i) an increase in price results in a decrease in demand i.e. if $\frac{dP}{P} = \text{positive}$, then $\frac{dQ}{Q} = \text{negative}$.ii) a decrease in price results in an increase in demand i.e. $\frac{dP}{P} = \text{negative}$, then $\frac{dQ}{Q} = \text{positive}$.Thus, $\frac{dP}{P}$ and $\frac{dQ}{Q}$ are of opposite signs i.e. one is positive and one is negative.

$$\text{So, } \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \text{negative} \quad \left[\text{and} \right] \quad \eta = -\frac{\frac{dQ}{Q}}{\frac{dP}{P}} = -(\text{neg}) = \text{positive}$$

 η is expected to be positive or $\eta > 0$.

Percentage change in demand and revenue when there is a percentage change in price

We know the revenue function $R = pq$ we

derive generally, $dR = qdp + pdq$

$$\Rightarrow \frac{dR}{R} = \frac{qdp + pdq}{R} \leftarrow \text{divide both sides by } R$$

$$\Rightarrow \frac{dR}{R} = \frac{qdp + pdq}{pq} ; R = pq$$

$$\Rightarrow \frac{dR}{R} = \frac{qdp}{pq} + \frac{pdq}{pq}$$

$$\Rightarrow \frac{dR}{R} = \frac{dp}{p} + \frac{dq}{q}$$

percentage change in Revenue = Percentage change in price
+ Percentage change in demand.

We have already seen that

$$\eta = - \frac{\frac{dq}{q}}{\frac{dp}{p}} \Rightarrow -\eta \frac{dp}{p} = \frac{dq}{q}$$

Substituting into

$$\frac{dR}{R} = \frac{dp}{p} + \frac{dq}{q} \Rightarrow \frac{dR}{R} = \frac{dp}{p} - \eta \frac{dp}{p}$$

$$\Rightarrow \frac{dR}{R} = (1 - \eta) \frac{dp}{p}$$

Also, since p can be expressed in terms of a single variable q and vice versa, we have

$$\boxed{\frac{dq}{dp} = \frac{1}{\frac{dp}{dq}}} \quad \text{or} \quad \frac{dp}{dq} = \frac{1}{\frac{dq}{dp}}$$

Recap.

if $\eta > 1$, then demand is inelastic.

if $\eta = 1$, then demand is unit elastic

if $\eta < 1$, then demand is price elastic.

At unit elasticity, revenue is maximized.

Percentage change in price $\frac{dp}{p} = \left(\frac{\text{new price} - \text{old price}}{\text{old price}} \right) (100\%)$

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2 p 172 Find M

$$q = \sqrt{800 - p^2} \quad \text{at } p = \$20$$

$$M = -\frac{p}{q} \frac{dq}{dp}$$

$$\text{at } p = \$20, \quad q = \sqrt{800 - 20^2} = \sqrt{800 - 400} = \sqrt{400} = 20$$

$$\text{from } q = \sqrt{800 - p^2}$$

$$\Rightarrow \frac{dq}{dp} = \frac{1}{2\sqrt{800 - p^2}} (800 - p^2)' = \frac{1}{2\sqrt{800 - p^2}} (-2p)$$

$$\Rightarrow \frac{dq}{dp} = \frac{-p}{\sqrt{800 - p^2}}$$

$$\text{At } p = \$20, \quad \frac{dq}{dp} = \frac{-20}{\sqrt{800 - 20^2}} = \frac{-20}{20} = -1$$

$$\therefore M = -\frac{p}{q} \frac{dq}{dp} = -\frac{20}{20} (-1) = 1$$

6 p 172

$$p = -0.2q + 80 \quad \text{at } p = \$10$$

$$M = -\frac{p}{q} \frac{dq}{dp}$$

$$\text{when } p = \$10, \quad 10 = -0.2q + 80$$

$$\Rightarrow 0.2q = 80 - 10 = 70$$

$$\Rightarrow q = \frac{70}{0.2} = 350$$

$$\text{From } p = -0.2q + 80$$

$$\Rightarrow \frac{dp}{dq} = -0.2 \quad \text{Since } p \text{ is in terms of a single variable } q,$$

$$\frac{dq}{dp} = \frac{1}{\frac{dp}{dq}} = \frac{1}{-0.2} = -5$$

$$\therefore M = -\frac{p}{q} \frac{dp}{dq} = -\frac{10}{350} (-5) = \frac{1}{7}$$

demand is price inelastic

11 p 173

$$q = -0.5p^2 - 6p + 30$$

$$\text{Elasticity, } M = -\frac{p}{q} \frac{dq}{dp}$$

$$\text{from } q = -0.5p^2 - 6p + 30$$

$$\Rightarrow \frac{dq}{dp} = -p - 6, \quad \text{plugging into } M = -\frac{p}{q} \frac{dq}{dp}$$

$$\Rightarrow M = -\frac{p}{-0.5p^2 - 6p + 30} (-p - 6) = 1$$

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$$\Rightarrow -p(-p-6) = -0.5p^2 - 6p + 30$$

$$\Rightarrow p^2 + 6p = -0.5p^2 - 6p + 30$$

$$\Rightarrow 1.5p^2 + 12p - 30 = 0$$

$$\Rightarrow 3p^2 + 24 - 60 = 0$$

$$\Rightarrow (3p + 30)(p - 2) = 0$$

$$\Rightarrow 3p + 30 = 0 \quad \text{or} \quad p - 2 = 0$$

$$\Rightarrow p = -10 \quad \text{or} \quad p = 2$$

We reject $p = -10$ because $p \geq 0$. We accept $p = 2$ because $p = 2$ since $p \geq 0$.

$\therefore p = \$2$ will yield unit elasticity. The corresponding demand $q = -0.5(2)^2 - 6(2) + 30$

$$= -2 - 12 + 30 = 16. \text{ max revenue is } R = (2)(16) = \$32.$$

14p 173

$$\text{Given } 4p^2 - 50q^2 = 800$$

Currently $p = \$100$ per pair and $q = 28$ can be sold

i) Find M when $p = \$100$

$M = -\frac{p}{q} \frac{dq}{dp}$ Since we need $\frac{dq}{dp}$, we differentiate the demand function implicitly with respect to p .

$$\text{From } 4p^2 - 50q^2 = 800$$

$$\Rightarrow 8p \frac{dp}{dp} - 100q \frac{dq}{dp} = 0$$

$$\text{When } p = 100 \text{ and } q = 28$$

$$8(100)(1) - 100(28) \frac{dq}{dp} = 0$$

$$\Rightarrow 800 - 2800 \frac{dq}{dp} = 0$$

$$\Rightarrow 2800 \frac{dq}{dp} = 800$$

$$\Rightarrow \frac{dq}{dp} = \frac{800}{2800} = \frac{8}{28}$$

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$$\eta = -\frac{p}{q} \frac{dq}{dp} = \frac{100}{28} \left(\frac{2}{7} \right) = -\frac{100}{98} = \frac{50}{49} < 1$$

Demand is price inelastic.

ii) $\eta = -\frac{\frac{dq}{q}}{\frac{dp}{p}}$ New price = \$125.

$$\text{If } \frac{dp}{p} = \frac{\text{new} - \text{old}}{\text{old}} (100\%) = \frac{125 - 100}{100} (100) = 25\%$$

$$\text{So, } -\frac{50}{49} = -\frac{\frac{dq}{q}}{25\%}$$

$$\Rightarrow \frac{dq}{q} = \left(\frac{50}{49} \right) (25\%) = \frac{1250}{49} \approx 25.51\%$$

$$\text{and } \frac{dR}{R} = \frac{dp}{p} + \frac{dq}{q} = 25\% + 25.51\% = 50.51\%$$

iii) New price = \$80, so, $\frac{dp}{p} = \frac{\text{new} - \text{old}}{\text{old}} (100\%)$
 $= \frac{80 - 100}{100} (100\%) = -20\%$

$$\text{So, } -\frac{50}{49} = \frac{-\frac{dq}{q}}{-20\%}$$

$$\Rightarrow \frac{dq}{q} = -\frac{50}{49} (20) = -\frac{1000}{49} = -20.4082\%$$

$$\text{and } \frac{dR}{R} = \frac{dp}{p} + \frac{dq}{q} = -20\% - 20.4082\%$$

$$= -40.4082\%$$